Magnetic flux periodicity in a hollow *d*-wave superconducting cylinder

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(Received 6 January 2009; revised manuscript received 2 February 2010; published 25 February 2010)

The magnetic flux dependence of order parameter and supercurrent is studied in a hollow d-wave superconducting cylinder. It is shown that the existence of line nodal quasiparticles in a pure $d_{x^2-y^2}$ pairing state gives rise to an hc/e periodicity in the order parameter. We demonstrate that the flux periodicity in the supercurrent is sensitive to the detailed electronic band structure and electron filling factor. In particular, we find that the hc/e component in the current spectrum can be tuned by these parameter values together with the cylinder circumference. A similar study of a $d_{x^2-y^2}+id_{xy}$ pairing state verifies the peculiarity of unconventional superconductors with nodal structure.

DOI: 10.1103/PhysRevB.81.054521 PACS number(s): 74.20.Rp, 73.23.Ra, 74.25.Uv, 74.78.Na

I. INTRODUCTION

A fundamental property of all known superconductors is the formation of Cooper pairs[1] in the superconducting state. A far-reaching implication of this fact is the quantization of magnetic flux in units of $hc/2e$ in multiply connected superconducting geometries. The $hc/2e$ flux quantization has been used as a proof of electron pairing nature of both conventional[2–5] and high-temperature[6] superconductors in the superconducting state. Other related phenomena include the quantum oscillation in the transition temperature[7] and magnetic vortices each carrying an $hc/2e$ flux quantum.[8,9]

Quantum mechanically, there is no fundamental reason why the minimal flux periodicity must be $hc/2e$ in a superconductor. The gauge invariance can only guarantee a fundamental period of $\Phi_0=hc/e$.[4,10] Only when all Cooper pairs move in the same group velocity, can a substantial $\Phi_0/2$ periodicity be obtained. The Φ_0 periodicity has been in mesoscopic conventional superconducting rings[11–14] due to the level discreteness and Landau depairing effect. More surprisingly, recent studies have shown a severe breaking of $hc/2e$ periodicity in a d-wave superconducting loop,[15–17] as a result of the Cooper-pair angular momentum selection for the existence of Doppler-shifted zero-energy states.

In this paper, by a systematic study of the supercurrent in a hollow d-wave superconducting cylinder, we show an hc/e magnetic periodicity in the d-wave order parameter and demonstrate that the flux periodicity in the supercurrent is sensitive to the detailed electronic band structure and electron-filling factor. In particular, we find that the breaking of $hc/2e$ periodicity in the case of d-wave pairing is closely related to the particle-hole symmetry in the normal-state band structure, which gives rise to the van Hove singularity. When the particle-hole symmetry point is avoided, the $hc/2e$ periodicity can be restored almost completely in the supercurrent.

The outline of the paper is as follows. In Sec. II, we recall the Bogoliubov-de Gennes (BdG) equation within a tight-binding model as a formalism for the flux dependence of the order parameter and supercurrent in a ring formed by a d-wave superconductor film. Within the given geometry, the analytical expressions for the order parameter, electron density, and the supercurrent are derived. In Sec. III, we present the numerical results and discuss the flux periodicity sensitive to the system size, electronic-structure parameters, and the pairing symmetry. A summary is given in Sec. IV.

II. BOGOLIUBOV-DE GENNES EQUATION AND ITS SOLUTION

To be specific, we consider a hollow d-wave superconducting cylinder, as schematically shown in Fig. 1. Experimentally, the cylinder can be formed by a high-temperature superconductor film with its normal perpendicular to the CuO_2 plane. A magnetic flux Φ threads the cylinder parallel to its axis and also to the crystal b axis of the CuO_2 plane. Due to the weak interlayer coupling, the problem can be reduced to one on an essentially two-dimensional system. We define the x and y axis to be perpendicular and parallel to the flux direction, respectively. This set up can also avoid the nucleation of Abrikosov vortices, which will complicate the analysis. By choosing a gauge, where the vector potential does not appear explicitly in the Hamiltonian, the BdG equations can be written as[18]

$$\sum_j \begin{pmatrix} \mathcal{H}_{ij} & \Delta_{ij} \\ \Delta_{ij}^* & -\mathcal{H}_{ij}^* \end{pmatrix} \begin{pmatrix} u_j^n \\ v_j^n \end{pmatrix} = E_n \begin{pmatrix} u_i^n \\ v_i^n \end{pmatrix} \tag{1}$$

subject to the flux-modified boundary condition

$$\begin{pmatrix} u_{i_x+N_x,i_y}^n \\ v_{i_x+N_x,i_y}^n \end{pmatrix} = \begin{pmatrix} e^{i2\pi\Phi/\Phi_0} & 0 \\ 0 & e^{-i2\pi\Phi/\Phi_0} \end{pmatrix} \begin{pmatrix} u_{i_x,i_y}^n \\ v_{i_x,i_y}^n \end{pmatrix}. \tag{2}$$

Here (u_i^n, v_i^n) with i denoting the coordinates (i_x, i_y) are the eigenfunctions corresponding to eigenvalues E_n. The single-particle Hamiltonian is given by

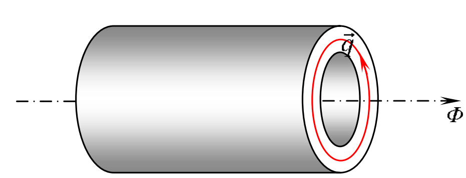

FIG. 1. (Color online) Schematic of a hollow d-wave superconducting cylinder. A magnetic flux threads the cylinder parallel to its axis.

$$\mathcal{H}_{ij} = -t_{ij} - \mu\delta_{ij}, \tag{3}$$

where t_{ij} and μ are the hopping integral and chemical potential, respectively. We consider the nearest-neighbor, t and next-nearest neighbor, t' hopping integral. The bond order parameter for d-wave pairing is determined self-consistently as

$$\Delta_{ij} = (V_{ij}/4)\sum_n [u_i^n v_j^{n*} + u_j^n v_i^{n*}]\tanh(E_n/2k_BT), \tag{4}$$

where the nearest-neighbor pairing interaction as defined by $V_{ij}=V_{x^2-y^2}$ is the pairing strength in the $d_{x^2-y^2}$ channel while the next-nearest-neighbor pairing interaction as defined by $V_{ij}=V_{nnn}=V_{xy}$ is the pairing strength in the d_{xy} channel. Notice that the quasiparticle excitation energy is measured with respect to the Fermi energy.

If the electron wave vector is $\mathbf{k}=(k_x,k_y)$ and that for the collective drift motion (superfluid motion) of the paired electrons is $\mathbf{q}=(q_x,q_y)$, the initial $\mathbf{k}$, and $-\mathbf{k}$ pairing is adjusted to pair the states $\mathbf{k}+\mathbf{q}$ and $-\mathbf{k}+\mathbf{q}$. The solution to the BdG equations is then found as

$$\begin{pmatrix} u_i \\ v_i \end{pmatrix} = \begin{pmatrix} e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}_i} & 0 \\ 0 & e^{i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}_i} \end{pmatrix} \begin{pmatrix} u_{\mathbf{k},\mathbf{q}} \\ v_{\mathbf{k},\mathbf{q}} \end{pmatrix}. \tag{5}$$

Here, corresponding to the eigenenergies

$$E^{(\pm)}_{\mathbf{k},\mathbf{q}} = Z_{\mathbf{k},\mathbf{q}} \pm E^{(0)}_{\mathbf{k},\mathbf{q}} \tag{6}$$

the electron and hole components of the quasiparticle amplitude are given by $(u_{\mathbf{k},\mathbf{q}},v_{\mathbf{k},\mathbf{q}})=(u^{(0)}_{\mathbf{k},\mathbf{q}},v^{(0)}_{\mathbf{k},\mathbf{q}})$ and $(v^{(0)*}_{\mathbf{k},\mathbf{q}},-u^{(0)*}_{\mathbf{k},\mathbf{q}})$ with

$$|u^{(0)}_{\mathbf{k},\mathbf{q}}|^2 = \frac{1}{2}\left(1+\frac{Q_{\mathbf{k},\mathbf{q}}}{E^{(0)}_{\mathbf{k},\mathbf{q}}}\right), \quad |v^{(0)}_{\mathbf{k},\mathbf{q}}|^2 = \frac{1}{2}\left(1-\frac{Q_{\mathbf{k},\mathbf{q}}}{E^{(0)}_{\mathbf{k},\mathbf{q}}}\right). \tag{7}$$

The quantities $Q_{\mathbf{k},\mathbf{q}}=[\xi_{\mathbf{k}+\mathbf{q}}+\xi_{\mathbf{k}-\mathbf{q}}]/2$, $Z_{\mathbf{k},\mathbf{q}}=[\xi_{\mathbf{k}+\mathbf{q}}-\xi_{\mathbf{k}-\mathbf{q}}]/2$, and $E^{(0)}_{\mathbf{k},\mathbf{q}}=[Q^2_{\mathbf{k},\mathbf{q}}+|\Delta_{\mathbf{k}}|^2]^{1/2}$. In the tight-binding approximation, up to the next-nearest neighbor, the conduction electrons have the normal-state dispersion

$$\xi_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - \mu. \tag{8}$$

The d-wave superconducting gap dispersion is given by

$$\Delta_{\mathbf{k}} = 2\Delta_{d_{x^2-y^2}}\phi_{x^2-y^2}(\mathbf{k}) + 2i\Delta_{d_{xy}}\phi_{xy}(\mathbf{k}), \tag{9}$$

where $\phi_{x^2-y^2}(\mathbf{k})=\cos k_x-\cos k_y$, $\phi_{xy}(\mathbf{k})=2\sin k_x \sin k_y$, and $\Delta_d(\mathbf{q})$ is determined self-consistently

$$\Delta_{d_{x^2-y^2}}(\mathbf{q}) = \frac{V_{x^2-y^2}}{4N_L}\sum_{\mathbf{k}} \frac{\phi^2_{x^2-y^2}(\mathbf{k})\Delta_{d_{x^2-y^2}}}{E^{(0)}_{\mathbf{k},\mathbf{q}}} \times \left[\tanh\left(\frac{E^{(0)}_{\mathbf{k},\mathbf{q}}+Z_{\mathbf{k},\mathbf{q}}}{2k_BT}\right)+\tanh\left(\frac{E^{(0)}_{\mathbf{k},\mathbf{q}}-Z_{\mathbf{k},\mathbf{q}}}{2k_BT}\right)\right], \tag{10}$$

$$\Delta_{d_{xy}}(\mathbf{q}) = \frac{V_{xy}}{4N_L}\sum_{\mathbf{k}} \frac{\phi^2_{xy}(\mathbf{k})\Delta_{d_{xy}}}{E^{(0)}_{\mathbf{k},\mathbf{q}}} \times \left[\tanh\left(\frac{E^{(0)}_{\mathbf{k},\mathbf{q}}+Z_{\mathbf{k},\mathbf{q}}}{2k_BT}\right)+\tanh\left(\frac{E^{(0)}_{\mathbf{k},\mathbf{q}}-Z_{\mathbf{k},\mathbf{q}}}{2k_BT}\right)\right], \tag{11}$$

where $N_L=N_x\times N_y$ and the quantities N_x and N_y represent the circumference and length of the hollow cylinder. Rigorously, the bond order parameters along the x and y directions do not follow the relation $\Delta_x=-\Delta_y$ in the presence of magnetic flux. Here we have imposed the restriction of $\Delta_x=-\Delta_y$ to enforce a rigorous d-wave symmetry. Notice that Δ_d is now a function of Φ/Φ_0. From the boundary condition given by Eq. (2), one can find[11] that the components of wave vectors $k_x=2\pi(n_x-m/2)/N_x$ and $q_x=2\pi(\Phi/\Phi_0+m/2)/N_x$ while $k_y=2\pi n_y/N_y$ and $q_y=0$, where $n_{x(y)}$ and m are integers. In particular, m is determined by minimizing $|q_x|$ for a given value of magnetic flux Φ. The electron filling factor and the single nearest-neighbor bond current flowing around the cylinder are computed via

$$n_e = \frac{2}{N_L}\sum_{\mathbf{k}} [f(E^{(+)}_{\mathbf{k},\mathbf{q}})|u^{(0)}_{\mathbf{k},\mathbf{q}}|^2 + f(E^{(-)}_{\mathbf{k},\mathbf{q}})|v^{(0)}_{\mathbf{k},\mathbf{q}}|^2], \tag{12}$$

$$I = \frac{4te}{N_L}\sum_{\mathbf{k}} [f(E^{(+)}_{\mathbf{k},\mathbf{q}})|u^{(0)}_{\mathbf{k},\mathbf{q}}|^2 + f(E^{(-)}_{\mathbf{k},\mathbf{q}})|v^{(0)}_{\mathbf{k},\mathbf{q}}|^2] \times (1+2t' \cos k_y/t)\sin(k_x+q_x), \tag{13}$$

respectively. A factor of 2 has been included to account for the spin degeneracy. The bond current as given in Eq. (13) is also equivalent to the energy derivation with respect to the flux, that is,

$$I = \frac{e}{N_L}\frac{\partial F}{\partial \varphi}, \tag{14}$$

where $\varphi=2\pi\Phi/N_x\Phi_0$, and the Helmholtz free energy, for a fixed electron-filling factor, is given by

$$F = -\frac{1}{\beta}\sum_{k=1}^{N_L}[\ln(1+e^{-\beta E^{(+)}_{\mathbf{kq}}(\Phi)}) + \ln(1+e^{-\beta E^{(-)}_{\mathbf{kq}}(\Phi)})] + \frac{4N_L}{V_{x^2-y^2}}|\Delta_{d_{x^2-y^2}}(\Phi)|^2 + \frac{4N_L}{V_{xy}}|\Delta_{d_{xy}}(\Phi)|^2 - N_L\mu(\Phi) + \bar{N}\mu(\Phi). \tag{15}$$

Here $\bar{N}\equiv\Sigma_{i,\sigma}\langle c^\dagger_{i,\sigma}c_{i,\sigma}\rangle=N_L n_e$ is the total average number of electrons.

III. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical calculations, we take $k_B=t=1$. Throughout the work, the energy is measured in units of t unless specified otherwise. The temperature is fixed at $T=0.01$ and the dominant $d_{x^2-y^2}$-wave channel pairing interaction is fixed at $V_{x^2-y^2}=4$. Both the hopping parameter t', the d_{xy}-wave

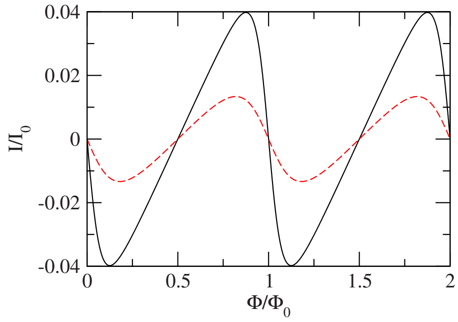

FIG. 2. (Color online) Flux dependence of the persistent current for a normal-state metallic cylinder of size $N_L=40^2$ (solid line) and $N_L=80^2$ (dashed line) with $n_e=1.0$ and $t'=0$. The current is measured in units of $I_0=et$. The other parameter values are defined in the main text.

channel pairing interaction, and the electron filling n_e will be changed. For a given n_e, the chemical potential should be adjusted and, therefore, will be a function of Φ/Φ_0.

In Fig. 2, we show the flux dependence of the persistent current in a normal state metallic cylinder, where there is no existence of superconducting order parameter. These results are known from the study of persistent current in normal-state mesoscopic rings[19] in the presence of an Aharonov-Bohm flux.[20] The main point is that the persistent current in a normal state ring has a periodicity of Φ_0. We present them here to demonstrate that our theoretical formulae designed for the superconducting state can reduce to describe the normal state and to provide a starting point for our discussion on the case of *d*-wave superconducting state below.

Figure 3 shows the flux dependence of the *d*-wave order parameter and persistent current for various electron filling and system size but with $t'=0$. Accordingly, we show the flux dependence of the free energy in Fig. 4. At the outset,

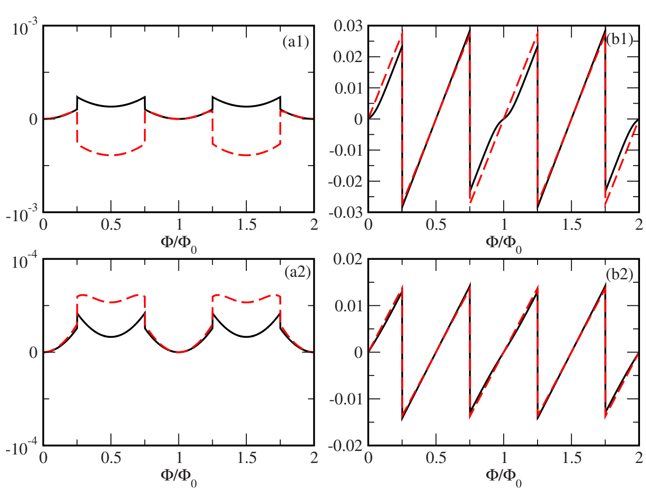

FIG. 3. (Color online) Flux dependence of the *d*-wave order parameter (a1-a2) and persistent current (b1-b2) for a *d*-wave superconducting cylinder of size $N_L=40^2$ (a1-b1) and $N_L=80^2$ (a2-b2) with $n_e=1.0$ (solid line) and $n_e=0.8$ (dashed line). In (a1-a2), the relative amplitude of the *d*-wave right parameter, $[\Delta_d(\Phi)-\Delta_d(\Phi=0)]/\Delta_d(\Phi=0)$, is shown. Here $t'=0$ and the other parameter values are defined in the main text.

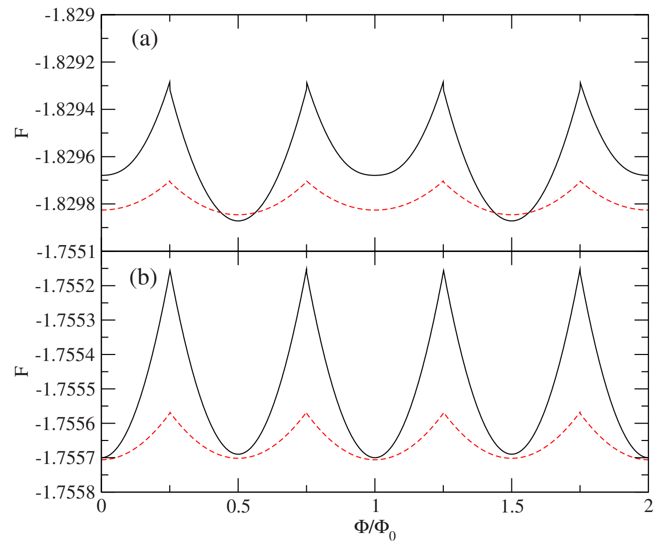

FIG. 4. (Color online) Free energy per site as a function of the magnetic flux for a superconducting cylinder for (a) $n_e=1$ and (b)$n_e=0.8$. The system sizes are chosen to be $N_L=40^2$ (black solid line) and $N_L=80^2$ (red dashed line). Here $t'=0$ and the other parameter values are defined in the main text.

we point out that the currents shown in the right panels of Fig. 3 are in good agreement with the numerical derivatives of the data in Fig. 4, as dictated by Eq. (14). For $T=0.01$ and $N_L=40\times40$, the zero-field order parameter $\Delta_{d_{x^2-y^2}}(\Phi=0)$ is about 0.620 for $n_e=1.0$ and 0.595 for $n_e=0.8$. The increase in the system size does not change the order parameter value much. Due to the existence of nodal quasiparticle in a $d_{x^2-y^2}$ pairing symmetry, the order parameter oscillates in the magnetic flux with a period of Φ_0, even for larger cylinders, although the oscillation amplitude is even smaller. The oscillation pattern is strongly sensitive to the system size. In particular, it is seen that the order parameter exhibits a discontinuous jump when the flux across $\Phi/\Phi_0=(2n-1)/4$ with n an integer. The jump amplitude also decreases with increased system size, indicating that this effect can only be observable in the mesoscopic regime. The Φ_0 periodicity in the pairing order parameter renders that the supercurrent does not develop fully the $\Phi_0/2$ periodicity, in a mathematically rigorous sense, regardless of the system size and electron filling. The reason lies in the fact that for a pure $d_{x^2-y^2}$-wave superconductor, low-energy quasiparticle states can be populated by the Doppler shift rapidly along the nodal direction, for which the orientation-dependent coherence length is divergent. As a hallmark of superconducting state, one can see clearly that flux-induced current has a different sign from that of the normal state in the region approximately from $\Phi/\Phi_0=n-1/4$ to $\Phi/\Phi_0=n+1/4$ [compare Figs. 3(b), 1, 3(b), and 2 with Fig. 2]. At the half filling, the particle-hole symmetry holds and the van Hove singularity emerges at the Fermi energy of the normal state. This makes a large level spacing at the Fermi energy accompanied by a more flattening of the low-lying quasiparticle energy dispersion near the Fermi energy for the flux close to $\mathrm{mod}(\Phi,\Phi_0)=0$, when the circumference of the cylinder is small. Correspondingly, one can notice from Fig. 4(a) that when the system size is small,

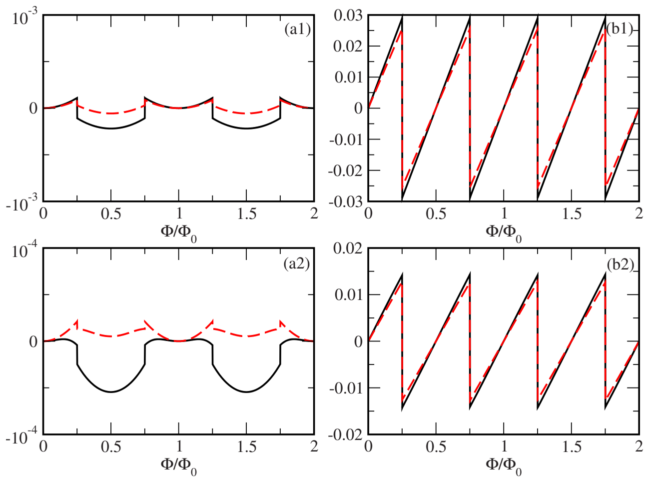

FIG. 5. (Color online) Flux dependence of the *d*-wave order parameter (a1-a2) and persistent current (b1-b2) for a *d*-wave superconducting cylinder. All the parameters of the figure are the same as those in Fig. 3 except that here $t'=-0.2$ while in Fig. 3 $t'=0$.

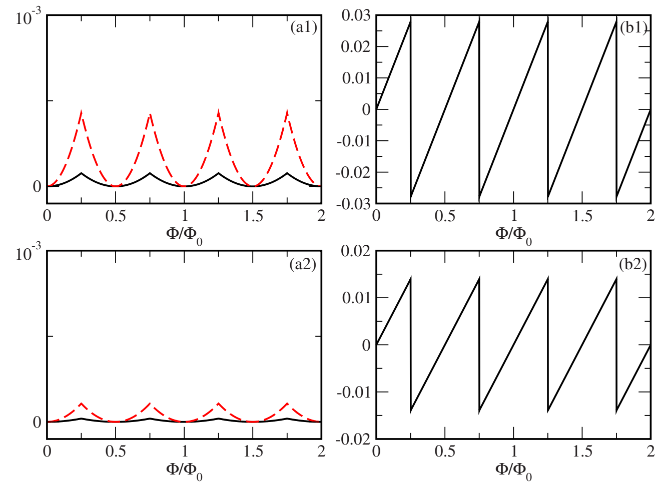

FIG. 6. (Color online) Flux dependence of the *d*-wave order parameter (a1-a2) and persistent current (b1-b2) for a $d_{x^2-y^2}+id_{xy}$-wave superconducting cylinder of size $N_L=40^2$ (a1-b1) and $N_L=80^2$ (a2-b2), with $n_e=1.0$. In (a1-a2), the relative amplitude of the respective $d_{x^2-y^2}$ (solid line) and d_{xy} (dashed line) components are plotted. Here $t'=0$ and the other parameter values are defined in the main text.

the free energy at $\Phi/\Phi_0=n$ is not only different in magnitude from but also flatter in curvature than that at $\Phi/\Phi_0=n\pm 1/2$. As such, the supercurrent exhibits an activation-like behavior for the magnetic flux close to $\Phi/\Phi_0=n$ and is very different from that at $\Phi/\Phi_0=n/2$. It explains why the Φ_0 periodicity of supercurrent is pronounced in this specific case. Furthermore, it is not difficult to anticipate that, *as a canonical mesoscopic effect*, the breaking of $hc/2e$ periodicity will be even more stronger for a smaller zero-field *d*-wave pair potential, and therefore a larger momentum-averaged superconducting coherence length $\xi=\langle \hbar v_{\mathbf{k}}/\pi\Delta_{\mathbf{k}}\rangle_{FS}$, where $v_{\mathbf{k}}$ is the quasiparticle velocity. We notice that, in the region $\Phi/\Phi_0\in[n-1/4,n+1/4]$, the flux dependence of the supercurrent in a *d*-wave superconducting cylinder is different from that in a square *d*-wave superconducting loop,[15] where a zigzag feature was obtained. We argue that in the geometry considered in Ref. 15, the elastic scattering from hard-wall boundaries of a mesoscale system populates a significant number of the lower-energy states, which plays an important role in the flux-dependent current. Naturally, the increase in the cylinder circumference is one way to reduce the activation behavior and therefore reducing the Φ/Φ_0 component in the Fourier spectrum of supercurrent. Alternatively, when the electronic filling factor is tuned away from the half filling, the particle-hole symmetry is broken and the Fermi surface becomes more isotropic. The activationlike behavior in the supercurrent is replaced by a more linearlike behavior, similar to the behavior exhibiting at $\Phi/\Phi_0=n\pm 1/2$. In addition, the current peaks at $\Phi/\Phi_0=(2n-1)/4$ becomes more symmetrical about $I=0$ axis, tending to restore the more of $\Phi_0/2$ periodicity in supercurrent.

Figure 5 shows the flux dependence of the *d*-wave order parameter and persistent current for various electron filling and system size but with $t'=-0.2$. For $T=0.01$ and $N_L=40\times 40$, the zero-field order parameter $\Delta_{d_{x^2-y^2}}(\Phi=0)$ is about 0.618 for $n_e=1.0$ and 0.617 for $n_e=0.8$. When a finite next-nearest-neighbor hopping integral is introduced, the particle-hole symmetry is broken at the outset for the hole doped (i.e., $n_e\leq 1$) system, where the chemical potential is tuned away from the van Hove singularity point. In this case, the level repulsion at the Fermi energy is weakened for flux close to $\Phi/\Phi_0=n$ even for a small-cylinder circumference, and the activation behavior in the current does not show up. Consequently, the Φ_0 component in the current spectrum is dramatically decreased, which makes the total current looks to have $\Phi_0/2$ periodicity completely. We note that when the zero-field-averaged superconducting coherence is not so small in comparison to the cylinder circumference, the restoration is always incomplete, due to the mesoscopic effect. The breaking of the $hc/2e$ periodicity with the hc/e now being the fundamental periodicity in the current is in support of the finding in Ref. 17 that the trapping of an $hc/2e$ vortex in a macroscopic superconducting ring is slightly more expensive in energy than the trapping of an hc/e vortex, when the superconducting material of the ring is of a unconventional pairing symmetry.

To understand better the magnetic flux periodicity in unconventional superconductors, we turn to consider a $d_{x^2-y^2}+id_{xy}$ pairing state by taking the pairing strength in the id_{xy} channel as $V_{xy}=3.0$. For $T=0.01$ and $N_L=40\times 40$, the zero-field order parameters $\Delta_{d_{x^2-y^2}}(\Phi=0)\approx 0.598$ and $\Delta_{d_{xy}}(\Phi=0)\approx 0.207$ for $n_e=1.0$. Now the quasiparticle excitations are gapful. In Fig. 6, we show the flux dependence of the *d*-wave order parameter and supercurrent in a hollow $d_{x^2-y^2}+id_{xy}$-wave superconducting cylinder. Noticeably, even in the presence of the particle-hole symmetry and for the same system size as the case of a pure $d_{x^2-y^2}$ pairing state, both the $d_{x^2-y^2}$ and d_{xy} components of order parameter have the periodicity of $\Phi_0/2=hc/2e$. In particular, one case see that the evolution of two components of order parameter is continuous when Φ/Φ_0 crosses $(2n-1)/4$, indicating that the flux-induced first-order quantum-phase transition is unique to a cylinder formed by unconventional superconductors with nodal quasiparticles. In the present case, the magnetic $hc/2e$ periodicity in the current is complete. The $hc/2e$ periodicity is set in as long as the cylinder circumference is much larger than the superconducting coherence length. We point out (but do not show) that the flux dependence of order parameter and supercurrent in a hollow *s*-wave superconducting cylinder exhibit similar behavior to the case of $d_{x^2-y^2}+id_{xy}$ pairing state.

One remark is in order: our calculations have shown that the magnitude of the order parameter can be enhanced in the presence of magnetic flux throughout the whole period of Φ_0 [see, e.g., Figs. 3(a), 2, 6(a), 1, 6(a), and 2]. This is the characteristic of all superconductors (including *s*-wave case) with short coherence length, which is much smaller than the cylinder circumference. We have calculated the case of a pure $d_{x^2-y^2}$-wave superconductor and a conventional *s*-wave superconductor both with a pairing interaction equal to 1 and found the flux dependence of the order parameter similar to that shown in Fig. 2 in Ref. 11 or Fig. 6 in Ref. 12, which again is a mesoscopic effect.

IV. SUMMARY

In conclusion, we have studied the flux dependence of order parameter and supercurrent in a hollow *d*-wave superconducting cylinder. For a pure $d_{x^2-y^2}$ pairing state, we find a hc/e periodicity of order parameter due to the existence of nodal quasiparticle states and an associated quantum-phase transition. When the particle-hole symmetry holds in the normal-state band structure, there is a noticeable component of hc/e in the supercurrent spectrum when the cylinder circumference in the mesoscopic regime. We find that, in addition to the system size, the relative weight of this component can also be tuned through the tuning of electron filling and band structure. By studying the case of a $d_{x^2-y^2}+id_{xy}$ pairing state, where the quasiparticle excitations are gapful, we verify that the peculiar hc/e magnetic flux periodicity only happens to unconventional superconductors with nodal structure.

ACKNOWLEDGMENTS

We thank Ye-Zheng Wu and Huai-Yu Duan for a technical help, I. Martin and Victor Vakaryuk for a useful discussion, and T. Kopp and Zlatko Tesanovic for correspondence. This work was carried out under the auspices of the National Nuclear Security Administration of the U.S. DOE at LANL under Contract No. DE-AC52-06NA25396, the LANL LDRD Programs, and the U.S. DOE Office of Science.

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